2

GENERALITIES

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The aim of this chapter is to present the student some generalities focusing on the atmospheric flight of airplanes. First, a classification of aerospace vehicles is given. Then, focusing on airplanes (which will be herein also referred to as aircraft), the main parts of an aircraft will be described. Third, the focus is on characterizing the atmosphere, in which atmospheric flight takes place. Finally, in order to be able to describe the movement of an aircraft, different system references will be presented. For a more detailed description of an aircraft, please refer for instance to any of the following books in aircraft design: TORENBEEK [8], HOWE [5], JENKINSON *et al.* [6], and RAYMER *et al.* [7].

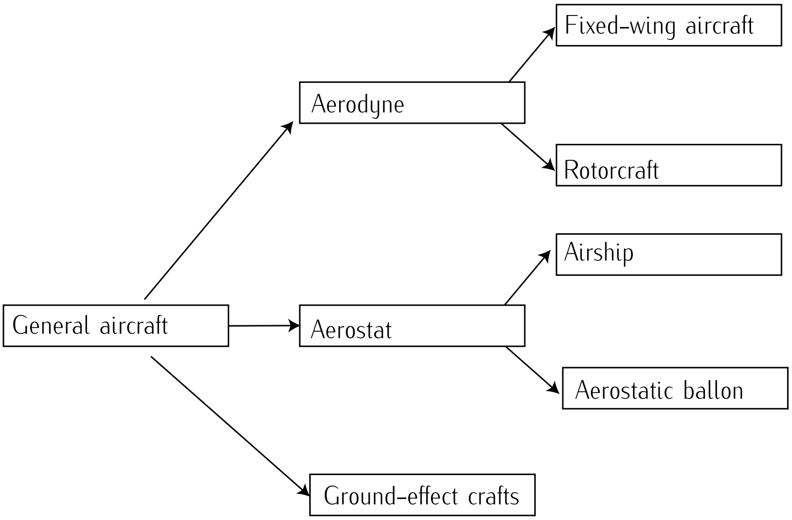


Figure 2.1: Classification of air vehicles. Adapted from FRANCHINI *et al.* [3].

2.1 CLASSIFICATION OF AEROSPACE VEHICLES (FRANCHINI *et al.* [3], FRANCHINI AND GARCÍA [2])

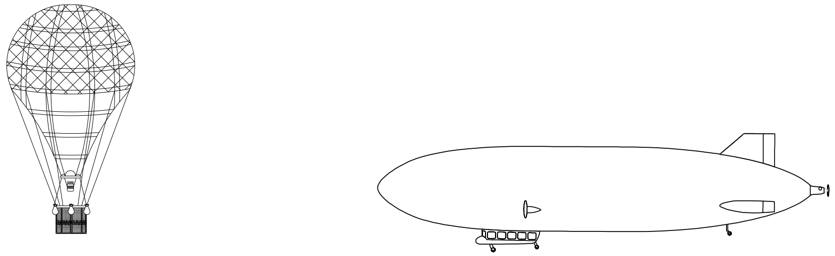
An aircraft, in a wide sense, is a vehicle capable to navigate in the air (in general, in the atmosphere of a planet) by means of a lift force. This lift appears due to two different physical phenomena:

- aeroestatic lift, which gives name to the aerostats (lighter than the air vehicles), and
- dynamic effects generating lift forces, which gives name to the aerodynes (heavier than the air vehicles).

An aerostat is a craft that remains aloft primarily through the use of lighter than air gases, which produce lift to the vehicle with nearly the same overall density as air. Aerostats include airships and aeroestatic balloons. Aerostats stay aloft by having a large "envelope" filled with a gas which is less dense than the surrounding atmosphere. See Figure 2.2 as illustration.

Aerodynes produce lift by moving a wing through the air. Aerodynes include fixed-wing aircraft and rotorcraft, and are heavier-than-the-air aircraft. The first group is the one nowadays know as airplanes (also known simply as aircraft). Rotorcraft include helicopters or autogyros (Invented by the Spanish engineer Juan de la Cierva in 1923).

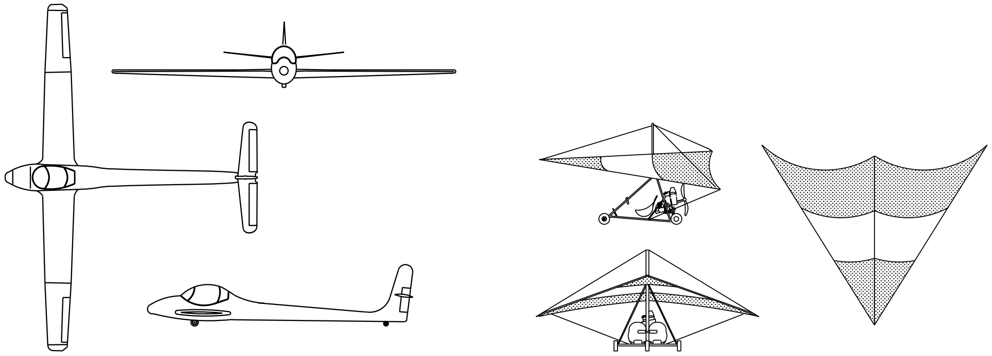
A special category can also be considered: *ground effect* aircraft. Ground effect refers to the increased lift and decreased drag that an aircraft airfoil or wing generates when an aircraft is close the ground or a surface. Missiles and space vehicles will be also analyzed as classes of aerospace vehicles.



(a) Aerostatic ballon.

(b) Airship (Zeppelin).

Figure 2.2: Aerostats.



(a) Glider.

(b) Hang glider (with engine).

Figure 2.3: Gliders.

2.1.1 FIXED WING AIRCRAFT

A first division arises if we distinguish those fixed-wing aircraft with engines from those without engines.

A glider is an aircraft whose flight does not depend on an engine. The most common varieties use the component of their weight to descent while they exploit meteorological phenomena (such thermal gradients and wind deflections) to maintain or even gain height. Other gliders use a tow powered aircraft to ascent. Gliders are principally used for the air sports of gliding, hang gliding and paragliding, or simply as leisure time for private pilots. See Figure 2.3.

Aerodynes with fixed-wing and provided with a power plant are known as airplanes[1]. An exhaustive taxonomy of airplanes will not be given, since there exist many particularities. Instead, a brief sketch of the fundamentals which determine the design of an aircraft will be drawn. The fundamental variables that must be taken into account for airplane design

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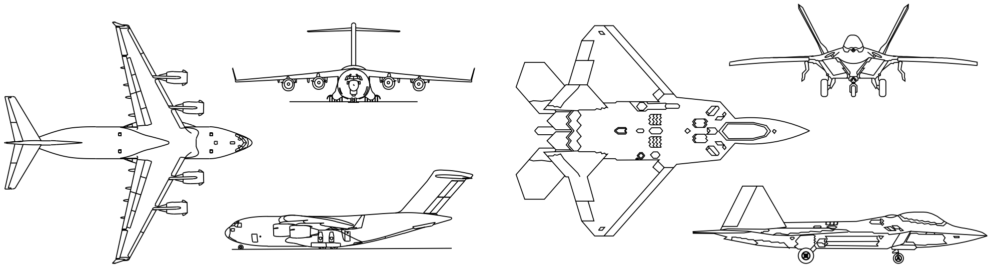
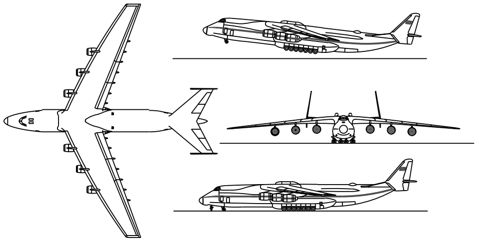
[1] Also referred to as aircraft. From now on, when we referred to an *aircraft*, we mean an aerodyne with fixed-wing and provided with a power plant.



(a) McDonnell Douglas MD-17 (military transportation).

(b) Lockheed Martin F22 Raptor (fighter).



(c) Antonov 225 (military transportation).

Figure 2.4: Military aircraft types.

are: mission, velocity range, and technological solution to satisfy the needs of the mission.

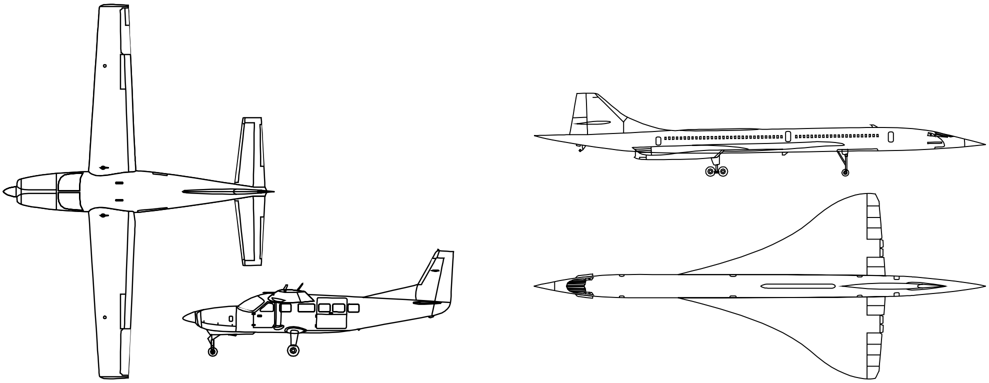
The configuration of the aircraft depends on the aerodynamic properties to fly in a determined regime (low subsonic, high subsonic, supersonic). In fact, the general configuration of the aircraft depends upon the layout of the wing, the fuselage, stabilizers, and power plant. This four elements, which are enough to distinguish, *grosso modo*, one configuration from another, are designed according to the aerodynamic properties.

Then one possible classification is according to its configuration. However, due to different technological solutions that might have been adopted, airplanes with the same mission, could have different configurations. This is the reason why it seems more appropriate to classify airplanes attending at its mission.

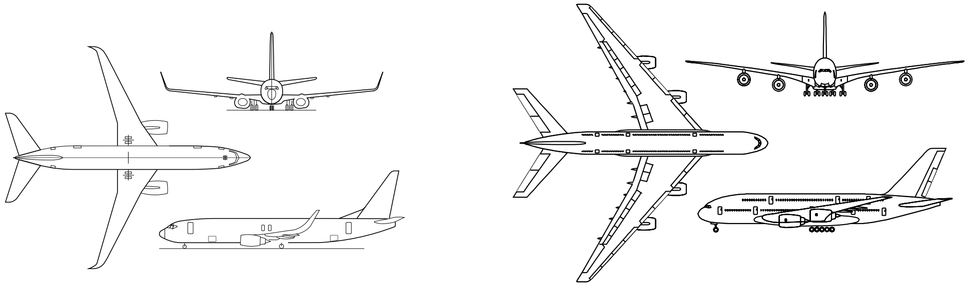
Two fundamental branches exist: military airplanes and civilian airplanes.

The most usual military missions are: surveillance, recognition, bombing, combat, transportation, or training. For instance, a combat airplane must flight in supersonic regime and perform sharp maneuvers. Figure 2.4 shows some examples of military aircraft.

In the civil framework, the most common airplanes are those dedicated to the transportation of people in different segments (business jets, regional transportation, medium-haul transportation, and long-haul transportation). Other civil uses are also derived to civil aviation such fire extinction, photogrametric activities, etc. Figure 2.5 shows some examples of civilian aircraft.



(a) Cessna 208 (Regional) (b) Concorde (Supersonic)



(c) Boeing 737-800 (Short-haul) (d) Airbus 380 (Long-haul)

Figure 2.5: Types of civilian transportation aircraft.

2.1.2 ROTORCRAFT

A rotorcraft (or rotary wing aircraft) is a heavier-than-air aircraft that uses lift generated by wings, called rotor blades, that revolve around a mast. Several rotor blades mounted to a single mast are referred to as a rotor. The International Civil Aviation Organization (ICAO) defines a rotorcraft as *supported in flight by the reactions of the air on one or more rotors*. Rotorcraft include:

- Helicopters.
- Autogyros.
- Gyrodinos.
- Combined.
- Convertibles.

A helicopter is a rotorcraft whose rotors are driven by the engine (or engines) during the flight, to allow the helicopter to take off vertically, hover, fly forwards, backwards, and laterally, as well as to land vertically. Helicopters have several different configurations

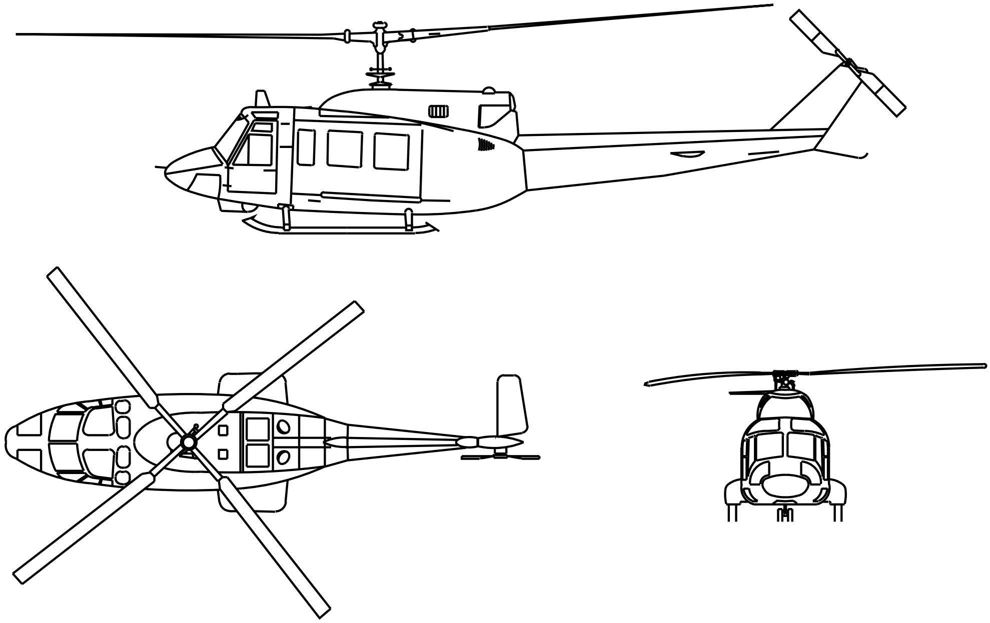


Figure 2.6: Helicopter.

of one or more main rotors. Helicopters with one driven main rotor require some sort of anti-torque device such as a tail rotor. See Figure 2.6 as illustration of an helicopter.

An autogyro uses an unpowered rotor driven by aerodynamic forces in a state of autorotation to generate lift, and an engine-powered propeller, similar to that of a fixed-wing aircraft, to provide thrust and fly forward. While similar to a helicopter rotor in appearance, the autogyro's rotor must have air flowing up and through the rotor disk in order to generate rotation.

The rotor of a gyrodyne is normally driven by its engine for takeoff and landing (hovering like a helicopter) with anti-torque and propulsion for forward flight provided by one or more propellers mounted on short or stub wings.

The combined is an aircraft that can be either helicopter or autogyro. The power of the engine can be applied to the rotor (helicopter mode) or to the propeller (autogyro mode). In helicopter mode, the propeller assumes the function of anti-torque rotor.

The convertible can be either helicopter or airplane. The propoller-rotor (proprotor) changes its attitude 90 [deg] with respect to the fuselage so that the proprotor can act as a rotor (helicopter) or as a propeller with fixed wings (airplane).

2.1.3 MISSILES

A missile can be defined as an unmanned self-propelled guided weapon system.

Missiles can be classified attending at different concepts: attending at the trajectory, missiles can be cruise, ballistic, or semi-ballistic. A ballistic missile is a missile that follows a sub-orbital ballistic flightpath with the objective to a predetermined target. The missile is only guided during the relatively brief initial powered phase of flight and its course is subsequently governed by the laws of orbital mechanics and ballistics. Attending at the target, missiles can be classified as anti-submarines, anti-aircraft, anti-missile, anti-tank, anti-radar, etc. If we look at the military function, missiles can be classified as strategic and tactical. However, the most extended criteria is as follows:

- Air-to-air: launched from an airplane against an arial target.
- Surface-to-air: design as defense against enemy airplanes or missiles.
- Air-to-surface: dropped from airplanes.
- Surface-to-surface: supports infantry in surface operations.

The general configuration of a missile consists in a cylindrical body with an ogival warhead and surfaces with aerodynamic control. Missiles also have a guiding system and are powered by an engine, generally either a type of rocket or jet engine.

2.1.4 SPACE VEHICLES

A space vehicle (also referred to as spacecraft or spaceship) is a vehicle designed for spaceflight. Space vehicles are used for a variety of purposes, including communications, earth observation, meteorology, navigation, planetary exploration, and transportation of humans and cargo. The main particularity is that such vehicles operate without any atmosphere (or in regions with very low density). However, they must scape the Earth's atmosphere. Therefore, we can identify different kinds of space vehicles:

- Artificial satellites.
- Space probes.
- Manned spacecrafts.
- Space launchers.

A satellite is an object which has been placed into orbit by human endeavor, which goal is to endure for a long time. Such objects are sometimes called artificial satellites to distinguish them from natural satellites such as the Moon. They can carry on board diverse equipment and subsystems to fulfill with the commended mission, generally to transmit data to Earth. A taxonomy can be given attending at the mission (scientific, telecommunications, defense, etc), or attending at the orbit (equatorial, geostationary, etc).

A space probe is a scientific space exploration mission in which a spacecraft leaves Earth and explores space. It may approach the Moon, enter interplanetary, flyby or orbit

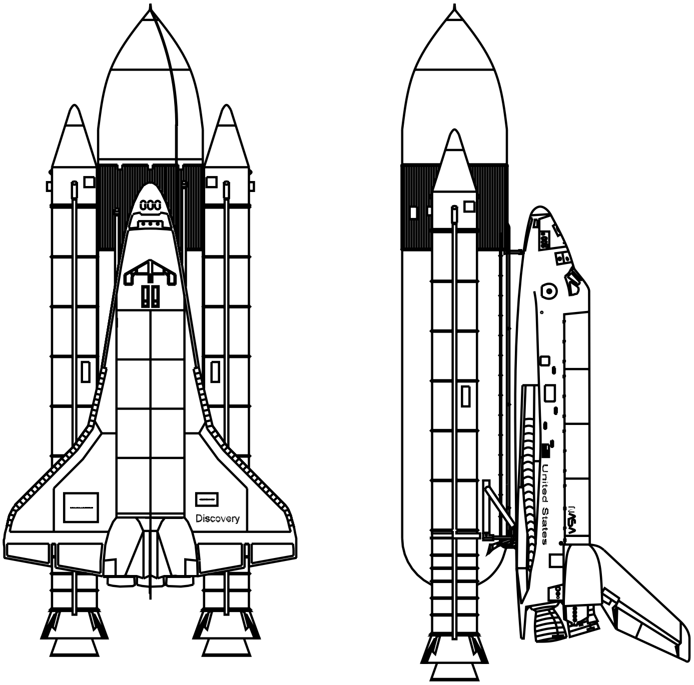


Figure 2.7: Space shuttle: Discovery.

other bodies, or approach interstellar space. Space probes are a form of robotic spacecraft. Space probes are aimed for research activities.

The manned spacecraft are space vehicles with crew (at least one). We can distinguish space flight spacecrafts and orbital stations (such ISS). Those missions are also aimed for research and observation activities.

Space launchers are vehicles which mission is to place another space vehicles, typically satellites, in orbit. Generally, they are not recoverable, with the exception of the the American space shuttles (Columbia, Challenger, Discovery, Atlantis, and Endevour). The space shuttle was a manned orbital rocket and spacecraft system operated by NASA on 135 missions from 1981 to 2011. This system combined rocket launch, orbital spacecraft, and re-entry spaceplane. See Figure 2.7, where the Discovery is sketched. Major missions included launching numerous satellites and interplanetary probes, conducting space science experiments, and 37 missions constructing and servicing the ISS.

The configuration of space vehicles varies depending on the mission and can be unique. As a general characteristic, just mention that launchers have similar configuration as missiles.

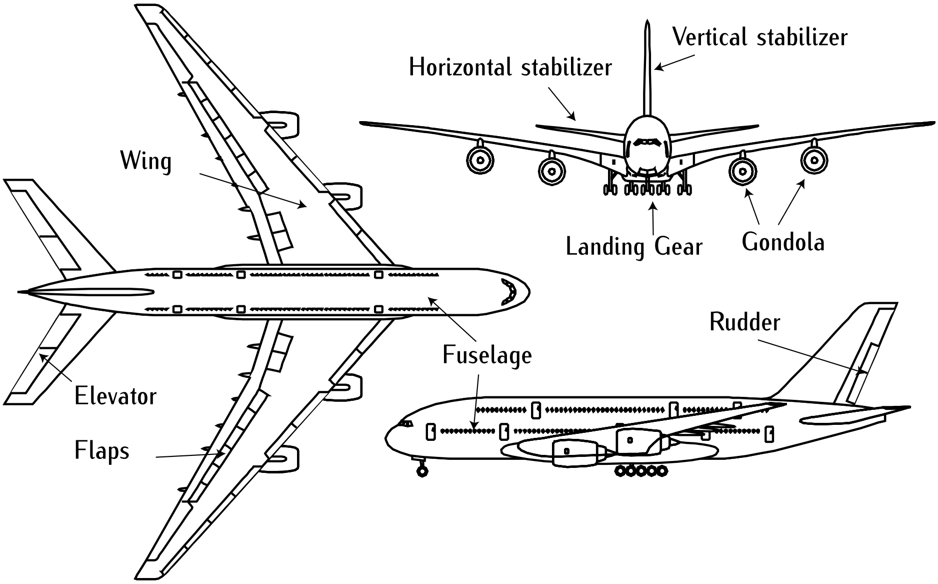


Figure 2.8: Parts of an aircraft.

2.2 PARTS OF THE AIRCRAFT (FRANCHINI *et al.* [3])

Before going into the fundamentals of atmospheric flight, it is interesting to identify the fundamental elements of the aircraft[2]. As pointed out before, there are several configurations. The focus will be on commercial airplanes flying in high subsonic regimes, the most common ones. Figure 2.8 shows the main parts of a typical commercial aircraft.

The central body of the airplane, which hosts the crew and the payload (passengers, luggage, and cargo), is the fuselage. The wing is the main contributor to lift force. The surfaces situated at the tail or empennage of the aircraft are referred to as horizontal stabilizer and vertical stabilizer. The engine is typically located under the wing protected by the so-called gondolas (some configurations with three engines locate one engine in the tail).

2.2.1 FUSELAGE

The fuselage is the aircraft's central body that accommodates the crew and the payload (passengers and cargo) and protect them from the exterior conditions. The fuselage also gives room for the pilot's cabin and its equipments, and serves as main structure to which the rest of structures (wing, stabilizers, etc.) are attached. Its form is a trade off between an aerodynamic geometry (with minimum drag) and enough volume to fulfill its mission.

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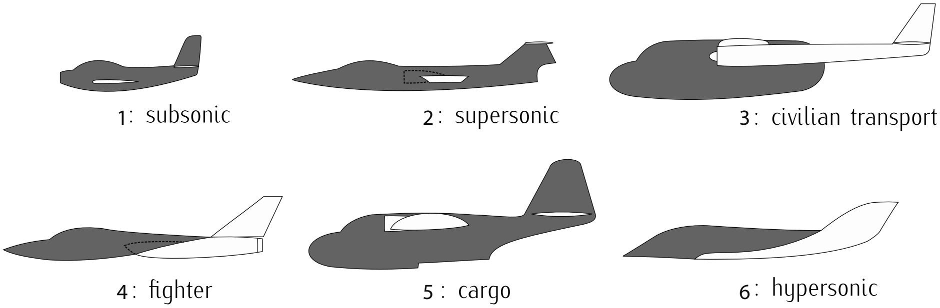
[2] Again, in the sense of a fixed-wing aircraft provided with a power plant.



1: subsonic

2: supersonic

3: civilian transport

4: fighter

5: cargo

6: hypersonic

Figure 2.9: Types of fuselages. © Adrián Hermida / Wikimedia Commons / CC-BY-SA-3.0.

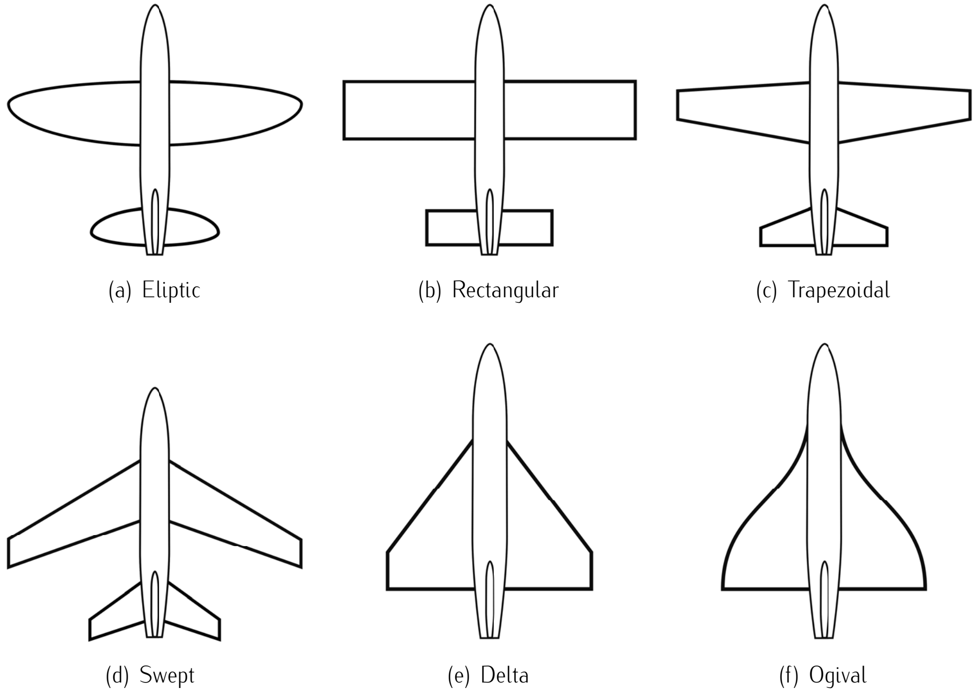
Most of the usable volume of the fuselage is derived to passenger transportation in the passenger cabin. The layout of the passenger cabin must fulfill IATA regulations (dimensions of corridors, dimensions of seats, distance between lines, emergency doors), and differs depending on the segment of the aircraft (short and long-haul), the passenger type (economic, business, first class, etc.), or company policies (low cost companies Vs. flag companies). Cargo is transported in the deck (in big commercial transportation aircraft generally situated bellow the passenger cabin). Some standardized types of fuselage are depicted in Figure 2.9.

2.2.2 Wing

A wing is an airfoil that has an aerodynamic cross-sectional shape producing a useful lift to drag ratio. A wing's aerodynamic quality is expressed as its lift-to-drag ratio. The lift that a wing generates at a given speed and angle of attack can be one to two orders of magnitude greater than the total drag on the wing. A high lift-to-drag ratio requires a significantly smaller thrust to propel the wings through the air at sufficient lift.

The wing can be classified attending at the plant-form. The elliptic plant-form is the best in terms of aerodynamic efficiency (lift-to-drag ration), but it is rather complex to manufacture. The rectangular plant-form is much easier to manufacture but the efficiency drops significantly. An intermediate solution is the wing with narrowing (also referred to as trapezoidal wing or tapered wing). As the airspeed increases and gets closer to the speed of sound, it is interesting to design swept wings with the objective of retarding the effects of sharpen increase of aerodynamic drag associated to transonic regimens, the so-called compressibility effects. The delta wing is less common, typical of supersonic flights. An evolution of the delta plant-form is the ogival plant-form. See Figure 2.10.

Attending at the vertical position, the wing can also be classified as high, medium, and low. High wings are typical of cargo aircraft. It allows the fuselage to be nearer the floor, and it is easier to execute load and download tasks. On the contrary, it is difficult to locate space for the retractile landing gear (also referred to as undercarriage). The


(a) Eliptic (b) Rectangular (c) Trapezoidal

(d) Swept (e) Delta (f) Ogival

Figure 2.10: Aircraft's plant-form types. © Guy Inchbald / Wikimedia Commons / CC-BY-SA-3.0.

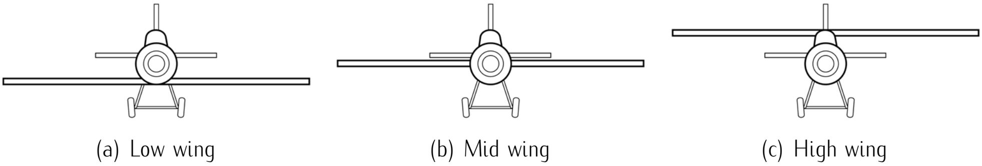

(a) Low wing (b) Mid wing (c) High wing

Figure 2.11: Wing vertical position. © Guy Inchbald / Wikimedia Commons / CC-BY-SA-3.0.

low wing is the typical one in commercial aviation. It does not interfere in the passenger cabin, diving the deck into two spaces. It is also useful to locate the retractile landing gear. The medium wing is not typical in commercial aircraft, but it is very common to see it in combat aircraft with the weapons bellow the wing to be dropped. See Figure 2.11.

Usually, aircraft's wings have various devices, such as flaps or slats, that the pilot uses to modify the shape and surface area of the wing to change its aerodynamic characteristics in flight, or ailerons, which are used as control surfaces to make the aircraft roll around its longitudinal axis. Another kind of devices are the spoilers which typically used to help braking the aircraft after touching down. Spoilers are deflected so that the lift gets

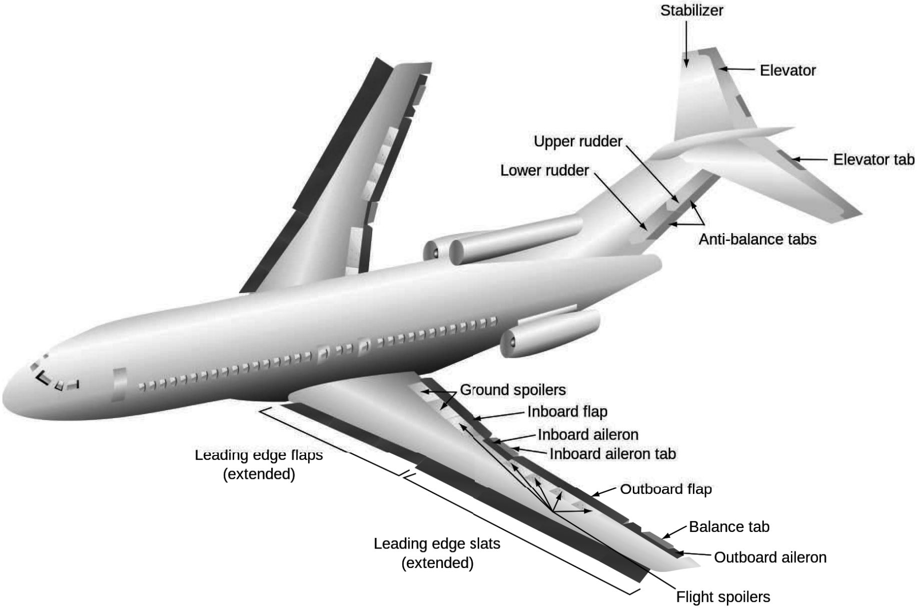


Figure 2.12: Wing and empennage devices. Wikimedia Commons / Public Domain.

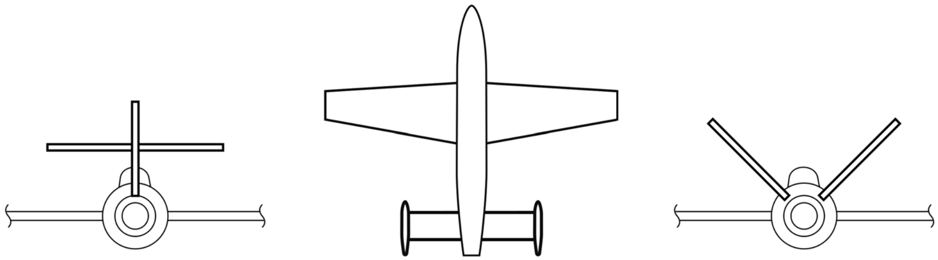
reduced in the semi-wing they are acting, and thus they can be also useful to help the aircraft rolling. If both are deflected at the same time, the total lift of the aircraft drops and can be used to descent quickly or to brake after touching down. See Figure 2.12.

2.2.3 EMPENNAGE

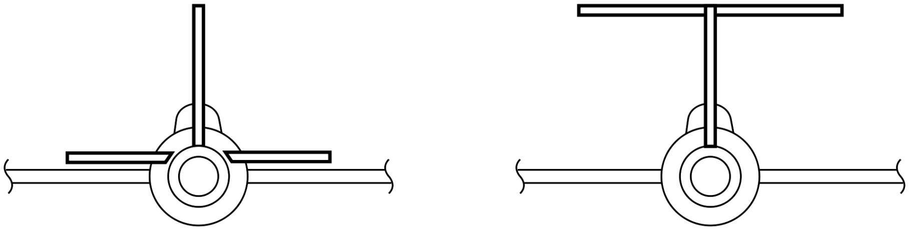
The empennage, also referred to as tail or tail assembly, gives stability to the aircraft. Most aircraft feature empennage incorporating vertical and horizontal stabilizing surfaces which stabilize the flight dynamics of pitch and yaw as well as housing control surfaces. Different configurations for the empennage can be identified (See Figure 2.13):

The conventional tail (also referred to as low tail) configuration, in which the horizontal stabilizers are placed in the fuselage. It is the conventional configuration for aircraft with the engines under the wings. It is structurally more compact and aerodynamically more efficient.

The cruciform tail, in which the horizontal stabilizers are placed midway up the vertical stabilizer, giving the appearance of a cross when viewed from the front. Cruciform tails are often used to keep the horizontal stabilizers out of the engine wake, while avoiding many of the disadvantages of a T-tail.



(a) Cruciform (b) H-tail (c) V-tail



(d) Conventional (e) T-tail

Figure 2.13: Aircraft's empennage types. © Guy Inchbald / Wikimedia Commons / CC-BY-SA-3.0.

The T-tail configuration, in which the horizontal stabilizer is mounted on top of the fin, creating a "T" shape when viewed from the front. T-tails keep the stabilizers out of the engine wake, and give better pitch control. T-tails have a good glide ratio, and are more efficient on low speed aircraft. However, T-tails are more likely to enter a deep stall, and is more difficult to recover from a spin. T-tails must be stronger, and therefore heavier than conventional tails. T-tails also have a larger cross section.

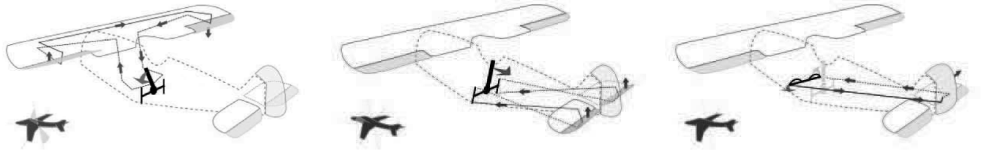
Twin tail (also referred to as H-tail) or V-tail are other configuration of interest although much less common.

2.2.4 MAIN CONTROL SURFACES

The main control surfaces of a fixed-wing aircraft are attached to the airframe on hinges or tracks so they may move and thus deflect the air stream passing over them. This redirection of the air stream generates an unbalanced force to rotate the plane about the associated axis.

The main control surfaces are: ailerons, elevator, and rudder.

Ailerons are mounted on the trailing edge of each wing near the wingtips and move in opposite directions. When the pilot moves the stick left, the left aileron goes up and



(a) Roll right (b) Pitch down (c) Yaw left

Figure 2.14: Actions on the control surfaces. © Ignacio Icke / Wikimedia Commons / CC-BY-SA-3.0.

the right aileron goes down. A raised aileron reduces lift on that wing and a lowered one increases lift, so moving the stick left causes the left wing to drop and the right wing to rise. This causes the aircraft to roll to the left and begin to turn to the left. Centering the stick returns the ailerons to neutral maintaining the bank angle. The aircraft will continue to turn until opposite aileron motion returns the bank angle to zero to fly straight.

An elevator is mounted on the trailing edge of the horizontal stabilizer on each side of the fin in the tail. They move up and down together. When the pilot pulls the stick backward, the elevators go up. Pushing the stick forward causes the elevators to go down. Raised elevators push down on the tail and cause the nose to pitch up. This makes the wings fly at a higher angle of attack, which generates more lift and more drag. Centering the stick returns the elevators to neutral position and stops the change of pitch.

The rudder is typically mounted on the trailing edge of the fin, part of the empennage. When the pilot pushes the left pedal, the rudder deflects left. Pushing the right pedal causes the rudder to deflect right. Deflecting the rudder right pushes the tail left and causes the nose to yaw to the right. Centering the rudder pedals returns the rudder to neutral position and stops the yaw.

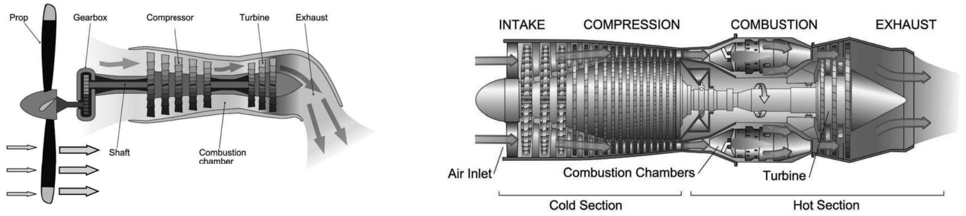
2.2.5 PROPULSION PLANT

The propulsion in aircraft is made by engines that compress air taken from the exterior, mix it with fuel, burn the mixture, and get energy from the resulting high-pressure gases.

There are two main groups: propellers and jets.

A propeller is a type of fan that transmits power by converting rotational motion into thrust. The first aircraft were propelled using a piston engine. Nowadays, piston engines are limited to light aircraft due to its weight and its inefficient performance at high altitudes. Another kind of propelled engine is the turbopropoller (also referred to as turboprop) engine, a type of turbine engine which drives an aircraft propeller using a reduction gear. Turboprop are efficient in low subsonic regimes.

A jet engine is a reaction engine that discharges a fast moving jet to generate thrust by jet propulsion in accordance with the third Newton's laws of motion (action-reaction).



(a) Turbopropoler © Emoscopes / Wikimedia Commons / CC-BY-SA-3.0.

(b) Jet engine © Jeff Dahl / Wikimedia Commons / CC-BY-SA-3.0.

Figure 2.15: Propulsion plant.

It typically consists of an engine with a rotating air compressor powered by a turbine (the so-called *Brayton cycle*), with the leftover power providing thrust via a propelling nozzle. This broad definition of jet engines includes turbojets, turbofans, rockets, ramjets, pulse jets. These types of jet engines are primarily used by jet aircraft for long-haul travel. Early jet aircraft used turbojet engines which were relatively inefficient for subsonic flight. Modern subsonic jet aircraft usually use high-bypass turbofan engines which provide high speeds at a reasonable fuel efficiency (almost as good as turboprops for low subsonic regimes).

2.3 Standard atmosphere

The International Standard Atmosphere (ISA) is an atmospheric model of how the pressure, temperature, density, and viscosity of the Earth's atmosphere change over a wide range of altitudes. It has been established to provide a common reference for the atmosphere consider standard (with an average solar activity and in latitudes around 45N). This model of atmosphere is the standard used in aviation and weather studies. The temperature of air is a function of the altitude, given by the profiles established by the International Standard Atmosphere (ISA) in the different layers of the atmosphere. The reader is referred, for instance, to Anderson [1] and Franchini and García [2] for a deeper insight.

2.3.1 Hypotheses

Hypothesis 2.1 (***Standard atmosphere***). *The basic hypotheses of ISA are:*

- *Complies with the perfect gas equation:*

$$p = \rho R T, \tag{2.1}$$

where R is the perfect gas constant for air (R=287.053 [J/kg K]), p is the pressure, ρ is the density, and T the temperature.

- *In the troposphere the temperature gradient is constant.*

$$\begin{aligned} &\textit{Troposphere } (0 \leq h < 11000\ [m]) : \\ &T = T_0 - \alpha h, \end{aligned} \tag{2.2}$$

where $T_0 = 288.15[K]$, $\alpha = 6.5[K/km]$.

- *In the tropopause and the inferior stratosphere the temperature is constant.*

$$\begin{aligned} &\textit{Tropopause and inferior stratosphere } (11000\ [m] \leq h < 20000\ [m]) : \\ &T = T_{11} \end{aligned} \tag{2.3}$$

where $T_{11} = 216.65[K]$.

- *The air pressure at sea level* ($h = 0$) *is* $p_0 = 101325$*[Pa]. In Equation* (2.1)*, the air density at sea level yields* $\rho_0 = 1.225[kg/m^3]$.
- *The acceleration due to gravity is constant* ($g = 9.80665[m/s^2]$)*.*
- *The atmosphere is in calm with respect to Earth.*

2.3.2 FLUID-STATIC EQUATION

Fluid statics (also called hydrostatics) is the science of fluids at rest, and is a sub-field within fluid mechanics. It embraces the study of the conditions under which fluids are at rest in stable equilibrium.

If we assume the air at rest as in Hypothesis (2.1), we can formulate the equilibrium of a differential cylindrical element where only gravitational volume forces and pressure surface forces act (see Figure 2.16):

$$pdS - (p + dp)dS = \rho g dS dh, \tag{2.4}$$

which gives rise to the equation of the fluid statics:

$$\frac{dp}{dh} = -\rho g. \tag{2.5}$$

2.3.3 ISA EQUATIONS

Considering Equation (2.1), Equations (2.2)-(2.3), and Equation (2.5), the variations of ρ and p within altitude can be obtained for the different layers of the atmosphere that affect atmospheric flight:

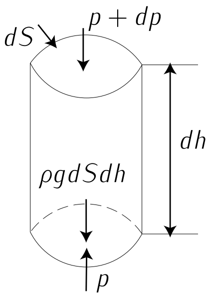


Figure 2.16: Differential cylinder of air. Adapted from FRANCHINI *et al.* [3].

Troposphere $(0 \leq h < 11000\ [m])$**:** Introducing Equation (2.1) and Equation (2.2) in Equation (2.5), it yields:

$$\frac{dp}{dh} = -\frac{p}{R(T_0 - \alpha h)} g. \tag{2.6}$$

Integrating between a generic value of altitude h and the altitude at sea level ($h = 0$), the variation of pressure with altitude yields:

$$\frac{p}{p_0} = (1 - \frac{\alpha}{T_0} h)^{\frac{g}{R\alpha}}. \tag{2.7}$$

With the value of pressure given by Equation (2.7), and entering in the equation of perfect gas (2.1), the variation of density with altitude yields:

$$\frac{\rho}{\rho_0} = (1 - \frac{\alpha}{T_0} h)^{\frac{g}{R\alpha} - 1}. \tag{2.8}$$

Introducing now the numerical values, it yields:

$$T[k] = 288.15 - 0.0065 h[m]; \tag{2.9}$$

$$\rho[kg/m^3] = 1.225(1 - 22.558 \times 10^{-6} \times h[m])^{4.2559}; \tag{2.10}$$

$$p[Pa] = 101325(1 - 22.558 \times 10^{-6} \times h[m])^{5.2559}. \tag{2.11}$$

Tropopause and Inferior part of the stratosphere $(11000\ [m] \leq h < 20000\ [m])$**:** Introducing Equation (2.1) and Equation (2.3) in Equation (2.5), and integrating between a generic altitude ($h > 11000$ [m]) and the altitude at the tropopause ($h_{11} = 11000$ [m]):

$$\frac{p}{p_{11}} = \frac{\rho}{\rho_{11}} = e^{-\frac{g}{RT_{11}}(h - h_{11})}. \tag{2.12}$$

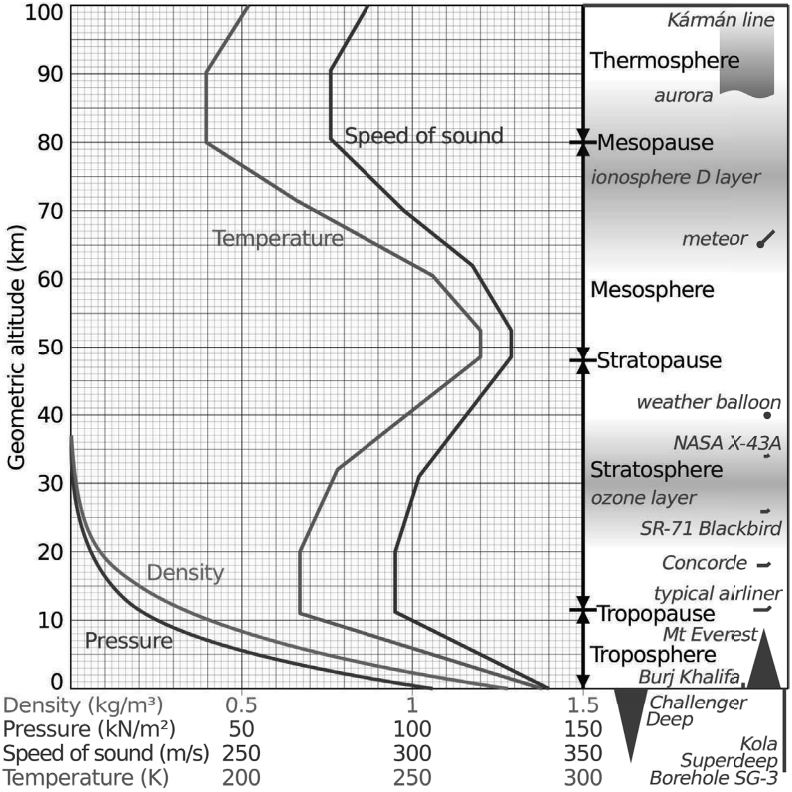


Figure 2.17: ISA atmosphere. © Cmglee / Wikimedia Commons / CC-BY-SA-3.0.

Introducing now the numerical values, it yields:

$$T[k] = 216.65; \tag{2.13}$$

$$\rho[kg/m^3] = 0.3639e^{-157.69 \cdot 10^{-6}(h[m]-11000)}; \tag{2.14}$$

$$p[Pa] = 22632e^{-157.69 \cdot 10^{-6}(h[m]-11000)}. \tag{2.15}$$

2.3.4 WARM AND COLD ATMOSPHERES

For warm and cold days, it is used the so called warm (ISA+5, ISA+10, ISA+15, etc.) and cold (ISA-5, ISA-10, ISA-15, etc.), where the increments (decrements) represent the difference with respect to the 288.15 [K] of an average day.

Given the new T_0, and given the same pressure at sea level (p_0 does not change), the new density at sea level can be calculated. Then the ISA equation are obtained proceeding in the same manner.

2.3.5 BAROMETRIC ALTITUDE

Altitude can be determined based on the measurement of atmospheric pressure. Still nowadays, most of the aircraft use the barometric altimeters to determine the altitude of the aircraft. An altimeter cannot, however, be adjusted for variations in air temperature. Differences in temperature from the ISA model will, therefore, cause errors in the indicated altitude.

Typically, an aneroid barometer or mercury barometer measures the atmospheric pressure from a static port outside the aircraft and based on a reference pressure. According to Equation (2.11) one has that:

$$\frac{p}{p_0} = (1 - \frac{\alpha}{T_0} h)^{\frac{g}{R\alpha}}; \qquad and \tag{2.16}$$

$$\frac{p_{ref}}{p_0} = (1 - \frac{\alpha}{T_0} h_{ref})^{\frac{g}{R\alpha}}. \tag{2.17}$$

Isolating h and h_{ref}, respectively, and subtracting, it yields:

$$h - h_{ref}[m] = \frac{T_0}{\alpha} \left[(\frac{p_{ref}}{p_0})^{\frac{R\alpha}{g}} - (\frac{p}{p_0})^{\frac{R\alpha}{g}} \right]. \tag{2.18}$$

The reference values can be adjusted, and there exist three main standards:

QNE setting: the baseline pressure is 101325 Pa. This setting is equivalent to the air pressure at mean sea level (MSL) in the International Standard Atmosphere (ISA).

QNH setting: the baseline pressure is the real pressure at sea level (not necessarily 101325 [Pa]). In order to estimate the real pressure at sea level, the pressure is measured at the airfield and then, using equation (2.8), the real pressure at mean sea level is estimated (notice that now $p_0 \neq 101325$). It captures better the deviations from the ISA.

QFE setting: where p_{ref} is the pressure in the airport, so that $h - h_{ref}$ reflects the altitude above the airport.

2.4 SYSTEM REFERENCES (GÓMEZ-TIERNO *et al.* [4])

The atmospheric flight mechanics uses different coordinates references to express the positions, velocities, accelerations, forces, and torques. Therefore, before going into the fundamentals of flight mechanics, it is useful to define some of the most important ones:

Definition 2.1 (***Inertial Reference Frame***). *According to classical mechanics, an inertial reference frame $F_I(O_I, x_I, y_I, z_I)$ is either a non accelerated frame with respect to a quasi-fixed reference star, or either a system which for a punctual mass is possible to apply the second Newton's law:*

$$\sum \vec{F_I} = \frac{d(m \cdot \vec{V_I})}{dt}$$

Definition 2.2 (***Earth Reference Frame***). *An earth reference frame* $F_e(O_e, x_e, y_e, z_e)$ *is a rotating topocentric (measured from the surface of the earth) system. The origin* O_e *is any point on the surface of earth defined by its latitude* θ_e *and longitude* λ_e*. Axis* z_e *points to the center of earth;* x_e *lays in the horizontal plane and points to a fixed direction (typically north);* y_e *forms a right-handed thrihedral (typically east).*

Such system is sometimes referred to as *navigational system* since it is very useful to represent the trajectory of an aircraft from the departure airport.

Hypothesis 2.2. Flat earth: *The earth can be considered flat, non rotating, and approximate inertial reference frame. Consider* F_I *and* F_e*. Consider the center of mass of the aircraft denoted by* CG*. The acceleration of* CG *with respect to* F_I *can be written using the well-known formula of acceleration composition from the classical mechanics:*

$$\vec{a}_I^{CG} = \vec{a}_e^{CG} + \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{r}_{O_I CG}) + 2\vec{\Omega} \wedge \vec{V}_e^{CG}, \tag{2.19}$$

where the centripetal acceleration ($\vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{r}_{O_I CG})$) *and the Coriolis acceleration* ($2\vec{\Omega} \wedge \vec{V}_e^{CG}$) *are neglectable if we consider typical values:* $\vec{\Omega}$ *(the earth angular velocity) is one revolution per day;* $\vec{r}$ *is the radius of earth plus the altitude (around 6380 [km]);* $\vec{V_e^{CG}}$ *is the velocity of the aircraft in flight (200–300 [m/s]). This means* $\vec{a}_I^{CG} \approx \vec{a}_e^{CG}$ *and therefore* F_e *can be considered inertial reference frame.*

Definition 2.3 (***Local Horizon Frame***). *A local horizon frame* $F_h(O_h, x_h, y_h, z_h)$ *is a system of axes centered in any point of the symmetry plane (assuming there is one) of the aircraft, typically the center of gravity. Axes* (x_h, y_h, z_h) *are defined parallel to axes* (x_e, y_e, z_e)*.*

In atmospheric flight, this system can be considered as quasi-inertial.

Definition 2.4 (***Body Axes Frame***). *A body axes frame* $F_b(O_b, x_b, y_b, z_b)$ *represents the aircraft as a rigid solid model. It is a system of axes centered in any point of the symmetry plane (assuming there is one) of the aircraft, typically the center of gravity. Axis* x_b *lays in to the plane of symmetry and it is parallel to a reference line in the aircraft (for instance, the zero-lift line), pointing forwards according to the movement of the aircraft. Axis* z_b *also lays in to the plane of symmetry, perpendicular to* x_b *and pointing down according to regular aircraft performance. Axis* y_b *is perpendicular to the plane of symmetry forming a right-handed thrihedral (*y_b *points then to the right wing side of the aircraft).*

Definition 2.5 (***Wind Axes Frame***). *A wind axes frame* $F_w(O_w, x_w, y_w, z_w)$ *is linked to the instantaneous aerodynamic velocity of the aircraft. It is a system of axes centered in any point of the symmetry plane (assuming there is one) of the aircraft, typically the center of gravity. Axis* x_w *points at each instant to the direction of the aerodynamic velocity of the aircraft* $\vec{V}$*. Axis* z_w *lays in to the plane of symmetry, perpendicular to* x_w *and pointing down according to regular aircraft performance. Axis* y_b *forms a right-handed thrihedral.*

Notice that if the aerodynamic velocity lays in to the plane of symmetry, $y_w \equiv y_b$.

2.5 PROBLEMS

Problem 2.1: International Standard Atmosphere

After the launch of a spatial probe into a planetary atmosphere, data about the temperature of the atmosphere have been collected. Its variation with altitude (h) can be approximated as follows:

$$T = \frac{A}{1+e^{\frac{h}{B}}}, \tag{2.20}$$

where A and B are constants to be determined.

Assuming the gas behaves as a perfect gas and the atmosphere is at rest, using the following data:

- *Temperature at* $h = 1000$, $T_{1000} = 250\ K$;
- $p_0 = 100000\frac{N}{m^2}$;
- $\rho_0 = 1\frac{Kg}{m^3}$;
- $T_0 = 300\ K$;
- $g = 10\frac{m}{s^2}$.

determine:

1. *The values of A and B, including their unities.*
2. *Variation law of density and pressure with altitude, respectively* $\rho(h)$ *and* $p(h)$ *(do not substitute any value).*
3. *The value of density and pressure at* $h = 1000\ m$.

Solution to Problem 2.1:

We assume the following hypotheses:

- a) *The gas is a perfect gas.*
- b) *It fulfills the fluidostatic equation.*

Based on hypothesis a):

$$P = \rho R T. \tag{2.21}$$

Based on hypothesis b):

$$dP = -\rho g dh. \tag{2.22}$$

Based on the data given in the statement, and using Equation (2.21)*:*

$$R = \frac{P_0}{\rho_0 T_0} = 333.3 \frac{J}{(Kg \cdot K)}. \tag{2.23}$$

1. *The values of A and B:*

   Using the given temperature at an altitude $h = 0$ ($T_0 = 300$ K), and Equation (2.20)*:*

$$300 = \frac{A}{1+e^0} = \frac{A}{2} \rightarrow A = 600\ K. \tag{2.24}$$

Using the given temperature at an altitude $h = 1000$ ($T_{1000} = 250$ K), and Equation (2.20)*:*

$$250 = \frac{A}{1+e^{\frac{1000}{B}}} = \frac{600}{1+e^{\frac{1000}{B}}} \rightarrow B = 2972\ m. \tag{2.25}$$

2. *Variation law of density and pressure with altitude:*

   Using Equation (2.21) *and Equation* (2.22)*:*

$$dP = -\frac{P}{RT} g dh. \tag{2.26}$$

Integrating the differential Equation (2.26) *between $P(h = 0)$ and P; $h = 0$ and h:*

$$\int_{P_0}^{P} \frac{dP}{P} = \int_{h=0}^{h} -\frac{g}{RT} dh. \tag{2.27}$$

Introducing Equation (2.20) *in Equation* (2.27)*:*

$$\int_{P_0}^{P} \frac{dP}{P} = \int_{h=0}^{h} -\frac{g(1+e^{\frac{h}{B}})}{RA} dh. \tag{2.28}$$

Integrating Equation (2.28)*:*

$$Ln\frac{P}{P_0} = -\frac{g}{RA}(h + Be^{\frac{h}{B}} - B) \rightarrow P = P_0 e^{-\frac{g}{RA}(h+Be^{\frac{h}{B}}-B)}. \tag{2.29}$$

Using Equation (2.21)*, Equation* (2.20)*, and Equation* (2.29)*:*

$$\rho = \frac{P}{RT} = \frac{P_0 e^{-\frac{g}{RA}(h+Be^{\frac{h}{B}}-B)}}{R\frac{A}{1+e^{\frac{h}{B}}}}. \tag{2.30}$$

3. *Pressure and density at an altitude of 1000 m:*

Using Equation (2.29) *and Equation* (2.30)*, the given data for* P_0 *and g, and the values obtained for R, A, and B:*

- $\rho(h = 1000) = 1.0756 \ \frac{kg}{m^3}$.
- $P(h = 1000) = 89632.5 \ Pa$.